

# Intrinsic Decoherence in Mesoscopic Systems

Maxim Vavilov and Vinay Ambegaokar

*Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca NY 14853, USA*

(September 20, 1997)

We point out that even at the absolute zero of temperature environmental decoherence limits the destructive interference between time-reversed paths for an electron in a disordered metal, and thus causes the leading ('weak localization') quantum correction to the conductivity to saturate at  $T = 0$ . Our calculation, which is intended to be illustrative rather than complete, uses a model in which an electron interacts with the fluctuations of the mean voltage in the sample. The average of the fluctuations produces the steady damping well known in Brownian motion, introduces a direction of time, ensures that arbitrarily long time-reversed electron paths lose phase coherence, and is consistent with the experimental observation of a finite low temperature phase-breaking lifetime.

PACS numbers: 72.15.-v, 72.70.+m, 73.23.-b

A recent experiment by Mohanty, Jariwala, and Webb [1] has called into question conventional wisdom about the first quantum correction to the electrical conductivity of disordered metals. This correction has been studied for 20 years and named weak localization [2]. The physical origin of the correction is the coherent back-scattering of electrons due to the quantum interference of time-reversed paths. The theory contains a phase-breaking lifetime which limits the length of destructively interfering paths. Calculations to date have this lifetime tending to infinity as the temperature approaches zero, but the experiments indicate that it remains finite.

In our opinion the previous theoretical work has not included the full effect of environmental fluctuations. We point out that the environment has an average dissipative effect, well known from the theory of Brownian motion, which survives at  $T = 0$  and appears to resolve the controversy. Our aim is to elucidate the missing physics using a simple model, leaving more detailed questions for further study.

We suppose that the electrons in the sample move in the electric field produced by fluctuations of the voltage in the electric circuit which contains the sample, and we consider the destruction of electron self-interference due to Nyquist noise at temperature  $T$ . We use the method of Feynman and Vernon [3] as elaborated by Leggett and Caldeira [4], and describe the electric field by a set of oscillators whose density is determined from the fluctuation-dissipation theorem. Averaging over the environmental oscillators yields the Feynman-Vernon influence functional which is the exponential of an action with both real and imaginary parts. For the purposes at hand, the real part plays the essential role; the contribution of the imaginary part—which in the classical limit is the effect of a fluctuating Langevin force [5]—to phase-breaking does indeed vanish at zero temperature.

The weak localization correction to the conductivity is, in a standard approach, proportional to an integral over time  $t$  of the probability  $P(t)$  that electron starting from

some point at  $t = 0$  returns to the same point at time  $t$ . We use this starting point, as developed by Chakravarty and Schmid [6], introducing the random impurity potential averaged over microscopic length scales and then integrating over the different quasi-classical trajectories.

At low temperatures the decoherence time due to Nyquist noise is the slowest function of temperature, and only one source of the destruction of the coherence. Besides, we limit ourselves here to the case of quasi one dimensional systems. Our result (see Eq.(17)) appears to apply to some of the data presented in [1], but it is worth mentioning again that we consider this to be a model calculation, designed to illustrate some essential physics. Important other sources of decoherence such the electron-electron and the electron-phonon [7,8] interaction have been omitted.

We consider a system described by the Hamiltonian

$$H = H_e + H_R + H_{e-R}, \quad (1)$$

where

$$H_e = \frac{p^2}{2M} \quad (2)$$

is the Hamiltonian of the free electron, and

$$H_R = \frac{1}{2} \sum_{\alpha} \left( \frac{p_{\alpha}^2}{m_{\alpha}} + m_{\alpha} \omega_{\alpha} x_{\alpha}^2 \right) \quad (3)$$

is the Hamiltonian of the environment, assumed to consist oscillators with a distribution to be given later. The interaction of the environment and the electron has the form

$$H_{e-R} = -q \frac{e}{L} \sum_{\alpha} x_{\alpha}. \quad (4)$$

The environmental degrees of freedom are the harmonics of the voltage across the sample. In Eq. (4)  $L$  is the length of the sample and  $q$  denotes the coordinate of the electron.

Because we are interested in the motion of the electron only we should average the density matrix over the environmental degrees of freedom. Then we get [3-5,9]

$$P(t) = \rho(q_0, q_0, t) =$$

$$\int_{q_0}^{q_0} \mathcal{D}q(\tau) \int_{q_0}^{q_0} \mathcal{D}q'(\tau) \exp \left( i(S_0[q] - S_0[q']) \right) F[q, q'], \quad (5)$$

where  $S_0$  is the action for a free particle,

$$F[q, q'] = \exp \left( iS_1[q, q'] - S_2[q, q'] \right) \quad (6)$$

is the influence functional, and  $S_1[q, q']$  and  $S_2[q, q']$  are the real and imaginary parts of the effective action,

$$S_1[q, q'] = \int_0^t ds \int_0^s du \sum_{\alpha} \frac{e^2}{2m_{\alpha}\omega_{\alpha}L^2} \times \sin(\omega_{\alpha}(s-u)) (q(s) - q'(s)) (q(u) + q'(u)), \quad (7)$$

$$S_2[q, q'] = \int_0^t ds \int_0^s du \sum_{\alpha} \frac{e^2}{2m_{\alpha}\omega_{\alpha}L^2} \coth \frac{\omega_{\alpha}}{2T} \times \cos(\omega_{\alpha}(s-u)) (q(s) - q'(s)) (q(u) - q'(u)). \quad (8)$$

The imaginary part  $S_2$  of the effective action contains the correlator of the environment field, quantum or classical, but in the classical case we have instead of the function  $\coth(\omega/2T)$ , its approximation at small values of the argument, i.e.  $2T/\omega$ . This approximation is appropriate when the temperature of the system is larger than the characteristic frequencies of the system. At lower temperature, the more accurate approximation is

$$\coth(\omega/2T) \approx \text{sign}\omega + 2T/\omega. \quad (9)$$

One can view the first term in this equation as coming from the zero-point motion of the oscillators. We shall see below that it makes no contribution to the return probability, thus calling into question the attribution in ref. [1] of the observed saturation to ‘zero point fluctuations.’ By contrast, the real part of the effective action does not depend on the temperature of the system and, as we shall see, it contributes to the zero temperature behavior of the system. This factor describes the average damping effect of the fluctuating field.

The summation over the environmental modes  $\alpha$  may be replaced by an integration over frequency of the density of modes of the voltage, defined by  $J(\omega) = 2\pi \sum_{\alpha} \delta(\omega - \omega_{\alpha})/2m_{\alpha}\omega_{\alpha}$ . This can in turn be determined from the fluctuation-dissipation theorem. The correlator of the voltage is  $\langle x(t)x(0) + x(0)x(t) \rangle / 2$ , where  $x(t) = \sum_{\alpha} x_{\alpha}(t)$ ; the average, with respect to the Gibbs distribution corresponding to Eq. (3), can be calculated explicitly. From the fluctuation dissipation theorem, the correlator is proportional to the susceptibility of the field,

here given by the impedance of the electric circuit which we take for simplicity to be purely resistive. As a result

$$J(\omega) = \omega R_{eff}. \quad (10)$$

We can now verify that the term  $\text{sign}\omega$  in Eq. (9) does not contribute to the motion by performing an integration over  $\omega$  in Eq. (8) from  $-\omega_c$  to  $\omega_c$ , where  $\omega_c$  is the cutoff [10]. Then we get a sharply peaked function of  $(s-u)$  whose integral over the difference  $(s-u)$  is exactly zero. [In performing this integral we may neglect the deviations of  $q(s)$  and  $q'(u)$ , since they may be considered slow functions of the difference  $(s-u)$  on the scale  $(\omega_c)^{-1}$ .] After these manipulations the imaginary part of the action is

$$S_2[q, q'] = \frac{e^2 T}{L^2} R_{eff} \int_0^t ds (q(s) - q'(s))^2. \quad (11)$$

With the choice (10) for  $J(\omega)$  the integral over modes can also be done for the real part of the action, yielding

$$S_1[q, q'] = -\frac{e^2}{2L^2} R_{eff} \int_0^t ds (q(s) + q'(s)) \times \frac{d}{ds} (q(s) - q'(s)), \quad (12)$$

where we have ignored a singular term proportional to  $\int_0^t ds (q^2(s) - q'^2(s))$  which makes no contribution to the time reversed paths considered below. [11]

Following Chakravarty and Schmid [6], we calculate the propagation of the electron in the field of random impurities, now, however, with the addition of the environmental interaction. We assume that there is a hierarchy of length scales  $\lambda_F \ll l_0 \ll l(T)$ , where  $\lambda_F$  is the electron wavelength,  $l_0$  is the impurity mean free path, and  $l(T)$  is the environmental decoherence length. We suppose that the motion between impurity collisions is described by the effective action given by Eqs. (5-12). Since this action is quadratic in the electron co-ordinate, it can in principle be done exactly [5], leaving a double sum over impurity positions. We argue as in ref. [6] that because of rapidly oscillating exponentials only time reversed paths contribute to this double sum, and then replace the single sum over trajectories by an average weighted by the probability of diffusive motion in the random impurity potential. It is now convenient to set the zero of time in such a way that the integration is over a symmetric interval. Then the relation between  $q(\tau)$  and  $q'(\tau)$  is  $q'(\tau) = q(-\tau)$ , with the weight of each trajectory given by  $\exp(-\int_{-t/2}^{t/2} dt q^2(\tau)/4D)$ , where  $D$  is the diffusion constant. It is convenient to introduce new variables  $x(\tau) = (q(\tau) + q(-\tau))/\sqrt{2}$  and  $\xi(\tau) = (q(\tau) - q(-\tau))/\sqrt{2}$ . Then the return probability is given by the expression

$$\begin{aligned}
P(t) = & \int_{-\infty}^{\infty} \frac{dx_0}{\sqrt{2}} \int_{x_0}^{x_f} \mathcal{D}x(\tau) \int_0^0 \mathcal{D}\xi(\tau) \times \\
& e^{-\frac{1}{4D} \int_0^{t/2} \dot{x}^2(\tau)} e^{-\frac{1}{4D} \int_0^{t/2} \dot{\xi}^2(\tau)} \times \\
& \exp \left( -4R_{eff} T \frac{e^2}{L^2} \int_0^{t/2} dt \xi^2(\tau) \right) \times \\
& \exp \left( -2iR_{eff} \frac{e^2}{L^2} \int_0^{t/2} dt \dot{\xi}(\tau) x(\tau) \right).
\end{aligned} \tag{13}$$

The integrals in this expression may be done successively. First we do the functional integral over  $x(\tau)$ . We find that the end points  $x_0$  and  $x_f$  then only occur in a Gaussian function of  $(x_0 - x_f - K)$  so that  $K$ , a constant depending on the path  $\xi(\tau)$ , and  $x_f$  drop out after a trivial integration over  $x_0$ . Finally one is left with a functional integral over  $\xi(\tau)$  equivalent to that for a harmonic oscillator. The final answer is

$$P(t) = \sqrt{\frac{\Omega}{8\pi D \sinh(\Omega t/2)}}, \tag{14}$$

with

$$\Omega^2 = 16 \left( \frac{D}{L^2} \frac{e^2 R_{eff}}{\hbar} \right) \left[ \frac{D}{L^2} \frac{e^2 R_{eff}}{\hbar} + \frac{T}{\hbar} \right]. \tag{15}$$

In the last equation the first (temperature independent) term comes from the real part of the action whereas the second comes from the imaginary part and vanishes at zero temperature ( $T \rightarrow 0$ ). At zero temperature, the driven response of the environment provides the friction [5] which breaks the symmetry with respect to time reflection and gives different contributions to the action of a trajectory and its time reflection.

The correction to the conductivity is the integral with respect to time of the return probability,

$$\Delta\sigma = -\frac{2e^2}{\pi\hbar} D \int_0^\infty dt P(t). \tag{16}$$

This integral converges at both limits at any temperature. Making the integral dimensionless we get

$$\Delta\sigma = -\alpha \frac{e^2}{\pi\hbar} \sqrt{\frac{D\hbar}{T_q}} \left( \frac{T_q}{T + T_q} \right)^{\frac{1}{4}}, \tag{17}$$

where

$$\alpha = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{d\xi}{\sqrt{\sinh \xi}} \approx 1.5 \tag{18}$$

and

$$T_q = \frac{D}{L^2} e^2 R_{eff}. \tag{19}$$

We see that there exists a characteristic temperature  $T_q$  below which the behavior of the system no longer depends on the environment temperature. In our model

this temperature is proportional to the inverse of the time it takes an electron to diffuse through the sample divided by the number of conducting channels. At high temperatures  $T \gg T_q$ , the correction to the conductivity Eq. (17) is in agreement with the result of ref. [6], and was observed in one dimensional samples in ref. [1].

To make connection with the usual expression for the correction to the conductivity in one dimension, we may define a quantity  $\tau(T)$  so that Eq. (17) takes the form

$$\Delta\sigma(T) = -\frac{e^2}{\pi\hbar} \sqrt{D\tau(T)}. \tag{20}$$

in the present model  $\tau(T)$ , which is usually referred to as the decoherence time of the electron system, is of the order  $\Omega^{-1}$ , with  $\Omega$  given by (15).

The expression for the  $\tau(T)$  has a universal form in the sense that it is completely described by one free parameter. If we choose this parameter to be the decoherence time at zero temperature, then

$$\tau(T) = \tau_0 \sqrt{\frac{1}{1 + T\tau_0/\alpha^2\hbar}}. \tag{21}$$

This behavior of  $\tau(T)$  is in qualitative accord with the experimental observations reported in [1] and references therein.

In summary, we have shown in a simple model that there is a temperature independent environmental decoherence effect which limits the backscattering of electrons in a disordered metal. We suggest that this is the essential physics behind the observed saturation of the weak localization correction to the conductivity.

## ACKNOWLEDGMENTS

This work is supported in part by the NSF under grant DMR-9407245.

- 
- [1] P.Mohanty, E.M.Q. Jariwala and R.A. Webb, Phys. Rev. Lett. **78**, 3366 (1997)
  - [2] B.L. Altshuler, A.G. Aronov, M.E. Gersenson and Yu. V. Sharvin, in *Soviet Scientific Reviews. Section A, Physics Reviews* edited by I.M.Khalatnikov ( Harwood Academic, New York, 1987), Vol. 9
  - [3] R.P. Feynman and F.L. Vernon, Ann. Phys. **24**, 118 (1963)
  - [4] A.O. Caldeira and A.J. Leggett, Ann. Phys. **149**, 374 (1983)

- [5] See, e.g., V. Ambegaokar in *Frontiers of Non Equilibrium Statistical Physics* edited by G.T.Moore and M.J. Scully (Plenum Press, 1986), p.235
- [6] Sudip Chakravarty and Albert Schmid, Phys.Rep, **140**, 193 (1986)
- [7] B.L. Altshuler and A.G. Aronov in *Electron–Electron Interactions in Disordered systems*, edited by A.L.Efros and M. Pollak (Elsevier, Amsterdam, 1985); B.L. Altshuler, A.G. Aronov and D.E. Khmelnitskii, J.Phys. C**15**, 7367 (1982)
- [8] B.L. Altshuler and A.G. Aronov Solid State Commun.**38**, 11 (1981)
- [9] V. Hakim and V. Ambegaokar, Phys. Rev. A, **32**, 423, (1985)
- [10] Our problem has the natural cutoff  $\omega_c \sim 1/\tau_0$ , where  $\tau_0$  is the elastic mean free time.
- [11] In the Leggett-Caldeira theory this singular piece is canceled by a counter term. Here, its presence or absence does not affect the physics